Solutions

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Indicate your final answer with a $\boxed{\text{box}}$.

1. Annual sales of music CDs have declined since 2000. Sales were 942.5 million in 2000 and 384.7 million in 2008.

   (a) (6 points) Find a formula for annual sales, S, in millions of music CDs, as a linear function of the number of years, t, since 2000.

   (b) (2 points) Give units for and intrepret the slope and the vertical intercept of this function.

   (c) (2 points) Use the formula to predict music CD sales in 2012.

(a) $\dfrac{384.7 - 942.5}{8} = -69.725$ $S(t) = 942.5 - 69.725t$

(b) vertical intercept at 942.5 million says that was sales in 2000.

(c) $S(12) = 942.5 - 69.725(12) = 105.8$ million

2. (5 points) A demand curve is given by $75p + 50q = 300$, where p is the price of the product, in dollars, and q is the quantity demanded at that price. Find p- and q-intercepts and interpret them in terms of consumer demand.

$75p = 300 \Rightarrow p = 4$ no product will be demanded at $4.

$50q = 300 \Rightarrow q = 6$ If the product is free, 6 of the product will be demanded.

3. A company produces and sells shirts. The fixed costs are \$7000 and the variable costs are \$5 per shirt.

(a) (4 points) Shirts are sold for \$12 each. Find cost and revenue as functions of the quantity of shirts, q.

(b) (4 points) The company is considering changing the selling price of the shirts. Demand is $q = 2000 - 40p$, where p is price in dollars and q is the number of shirts. What quantity is sold at the current price of \$12?

(c) (2 points) What profit is realized at this price of \$12?

(a) $\quad C = 5q + 7000 \qquad R = 12q$

(b) $\quad q(12) = 2000 - 40(12) = 1520$

(c) $\quad P = 7q - 7000 \implies P(1520) = \$8,640$

4. A product costs \$80 today. How much will the product cost in t days if the price is reduced by

(a) (2 points) \$4 a day

(b) (2 points) 5% a day

(a) $\quad C(t) = 80 - 4t$

(b) $\quad C(t) = 80(0.95)^t$

5. (6 points) The antidepressant Prozac has a half-life of 3 days. What percentage of a dose remains in one day?

$$\frac{1}{2} = e^{k \cdot 3} \implies k = \frac{\ln(\frac{1}{2})}{3} \approx -0.23$$

$$e^{-0.23} = 0.79 \implies 79\% \text{ of a dose}$$

6. In 1923, koalas were introduced on Kangaroo Island off the coast of Australia. In 1996, the population was 5000. By 2005, the population had grown to 27,000, prompting a debate on how to control their growth and avoid koalas dying of starvation. Assume the koala population grows at a exponential rate.

(a) (6 points) Find a formula for the population as a function of the number of years since 1996.

(b) (2 points) Estimate the population in the year 2020.

(c) (2 points) In what year is the population expected to exceed 100,000?

(a) $P = P(t) = 5000 \, e^{kt}$

$$P(9) = 27,000 = 5000 \, e^{k \cdot 9}$$

$$5.4 = e^{k9} \implies k = \frac{\ln(5.4)}{9} \approx 0.187$$

$$P(t) = 5000 \, e^{0.187t}$$

(b) $P(24) = 5000 \, e^{0.187(24)} \approx 444,717$

(c) $100,000 = 5000 \, e^{0.187t}$

$$20 = e^{0.187t} \implies t = \frac{\ln(20)}{0.187} \approx 16$$

So 2012

7. The island of Manhattan was sold for \$24 in 1626.

  (a) (5 points) How much money would be in the account in 2012 if the yearly interest rate was 5% compounded continuously?

  (b) (5 points) If the yearly interest rate was 6% compounded annually, in what year would the account be worth one billion dollars?

(a) In years since 1626 $A(t) = 24 e^{.05t}$

     In 2012, $A(386) = 5{,}782{,}221{,}743$

(b) $A(t) = 24 (1.06)^t$

$1{,}000{,}000{,}000 = 24(1.06)^t \Rightarrow t = 301.11$

So Year 1927.

8. Consider the functions $f(x) = 5x - 1$, $g(x) = e^{2x}$, and $h(x) = x^2 + 8$. Find the following:

  (a) (3 points) $f \circ g(x)$

  (b) (3 points) $h(x + 2)$

  (c) (3 points) $h(g(0))$

(a) $5e^{2x} - 1$

(b) $(x+2)^2 + 8$

(c) $h(g(0)) = h(1) = 9$

9. (4 points) Use the variable u for the inside function to express $P = \sqrt{5t^2 + 10}$ as a composite function.

$$P = \sqrt{u} \quad \text{where} \quad u = 5t^2 + 10$$

10. Consider the function $f(x) = \frac{1}{x}$.

(a) (6 points) Find the average velocity between $t = 2$ and $t = 2 + h$ if:

$$(i) \quad h = 0.1, \quad (ii) \quad h = 0.01, \quad (iii) \quad h = 0.001.$$

(b) (4 points) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time $t = 1$.

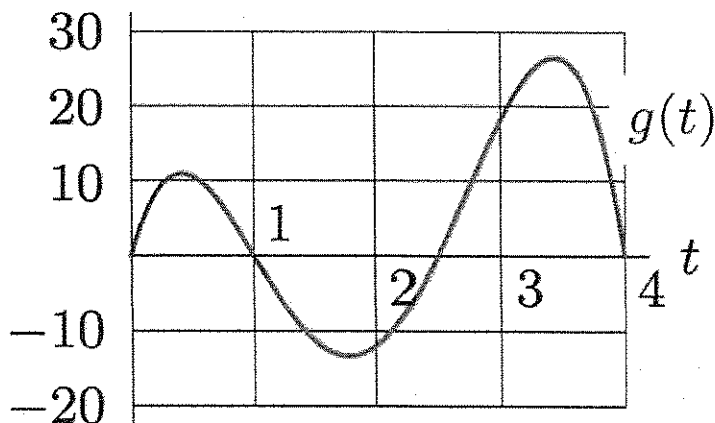(a) (i) $\dfrac{\frac{1}{2.1} - \frac{1}{2}}{.1} \approx -0.238$

(ii) $\dfrac{\frac{1}{2.01} - \frac{1}{2}}{.01} \approx -0.249$

(iii) $\dfrac{\frac{1}{2.001} - \frac{1}{2}}{.001} \approx -0.250$

(b) Outputs approach $-\frac{1}{4}$ as $h \to 0$ so

expect $f'(2) = -\frac{1}{4}$.

11. Consider the function g given below.

(a) (3 points) On what intervals is g positive?

(b) (3 points) On what intervals in g negative?

(c) (3 points) On what intervals is g' positive?

(d) (3 points) On what intervals in g' negative?

g positive: $(0,1), (2.5, 4)$

g negative: $(1, 2.5)$

g' positive: $(0, 0.4), (1.8, 3.4)$

g' negative: $(0.4, 1.8), (3.4, 4)$

Bonus Question: If you could meet one person in history, who would it be and why?

Isaac Newton because Calculus.